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## Physics-informed machine learning for SHM

Lizzy Cross, Daniel Pitchforth, Matthew Jones,  
Sam Gibson, Sikai Zhang, Tim Rogers



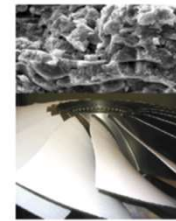
Dynamics  
Research  
Group

# About us

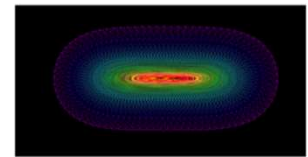
The **Dynamics Research Group** has been hosted by the Department of Mechanical Engineering at the University of Sheffield since 1995.

We are an established group with leading researchers in SHM, nonlinear dynamics, acoustics and damping. Current work on digital twins bridges these themes.

The group currently has over 50 members, including 12 academics and >40 post-doctoral researchers and PhD students.

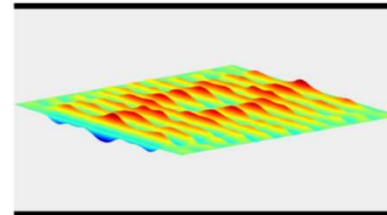


Damping



Nonlinear Dynamics

Acoustics



SHM



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# Road map

## Intro & motivation

- Motivation for a physics-informed approach to machine learning
- Research questions to be addressed

## Current work

- Examples of different model architectures and their uses

## Ongoing and future challenges

- Model validation
- Optimising balance between data and knowledge
- Conclusions: when are these methods useful?

### Publications

#### Overviews of grey-box models

Physics-informed machine learning for structural health monitoring  
E.J. Cross, S.J. Gibson, M.R. Jones, D.J. Pitchforth, S. Zhang, T.J. Rogers  
Book Chapter  
Structural Health Monitoring Based on Data Science Techniques, October 21  
DOI: [https://doi.org/10.1007/978-3-030-48776-9\\_17](https://doi.org/10.1007/978-3-030-48776-9_17)

#### Combining physics-based and machine learning models

Grey-box models for wave loading prediction  
D.J. Pitchforth, T.J. Rogers, U.T. Tjebben, E.J. Cross  
Mechanical Systems and Signal Processing, October 21  
DOI: <https://doi.org/10.1016/j.ymssp.2021.107794>  
Open access: <https://arxiv.org/abs/2105.13813>

Grey-Box Modelling via Gaussian Process Mean Functions for Mechanical Systems  
S. Zhang, T.J. Rogers, E.J. Cross  
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Gaussian Process Based Grey-Box Modelling for SHM of Structures Under Fluctuating Environmental Conditions  
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EWSHM 2022 European Workshop on Structural Health Monitoring, January 21  
DOI: [https://doi.org/10.1007/978-3-030-64908-1\\_6](https://doi.org/10.1007/978-3-030-64908-1_6)

#### Semi-physical modelling



[dr-greynbox.github.io/](https://dr-greynbox.github.io/)

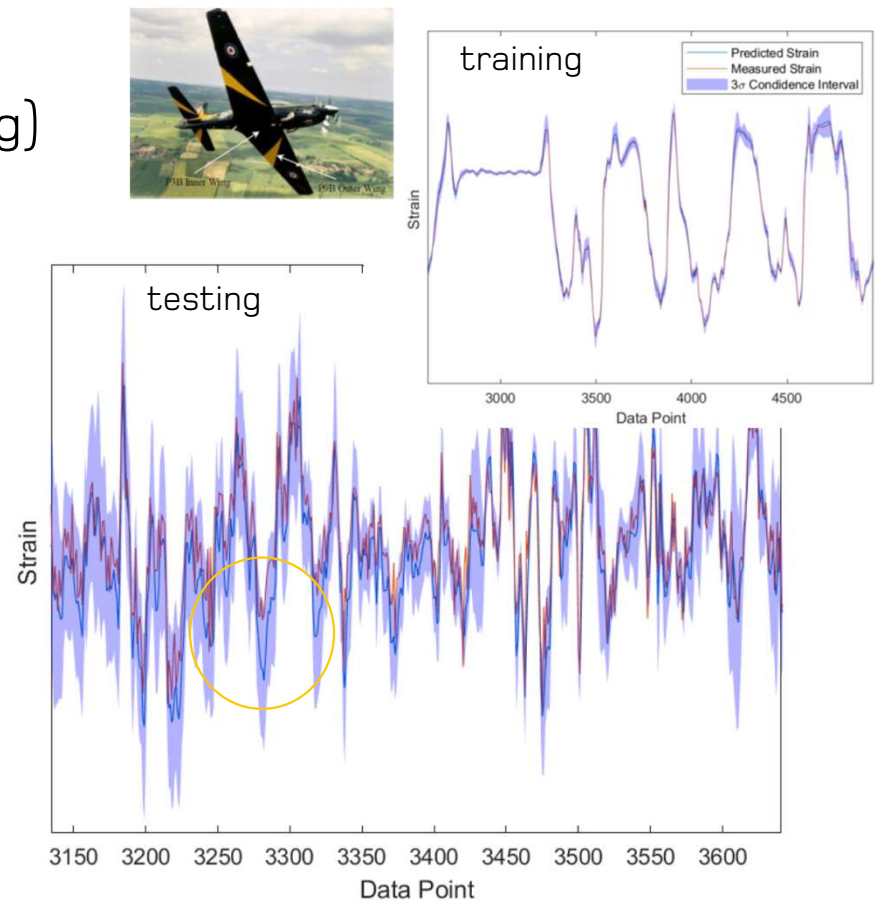


# Why are we interested in physics-informed ML?

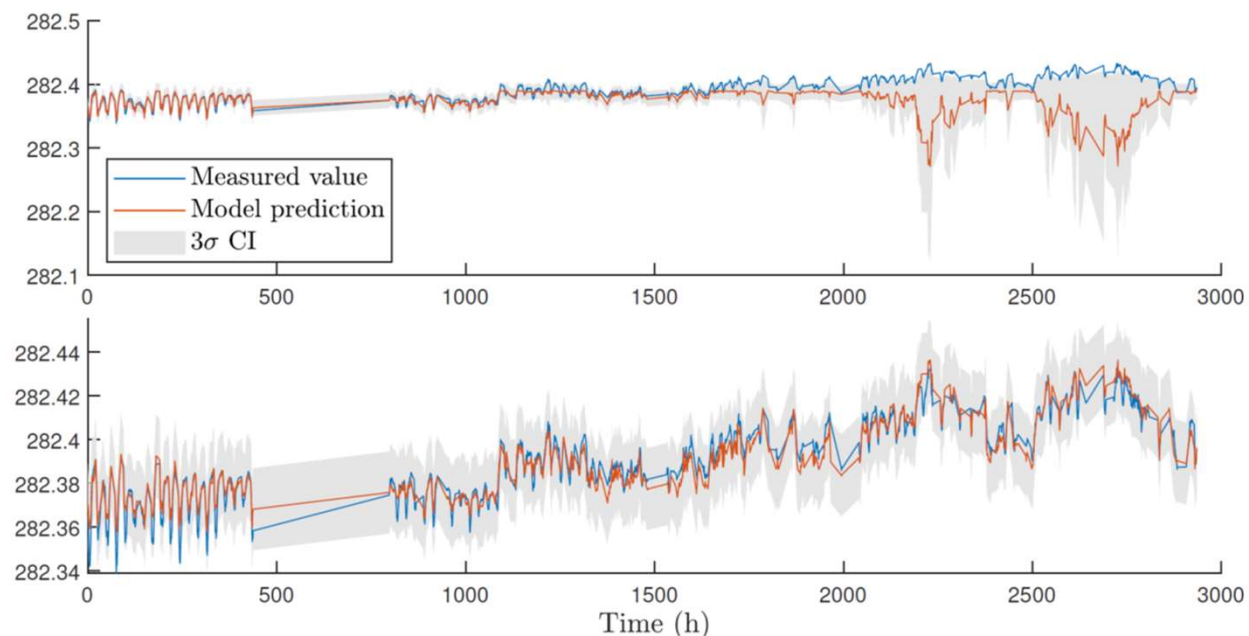
- We'd like to make inferences about our structures that are informed by (monitoring) data and our knowledge of the physics at work
- To address some of the challenges of using ML in an engineering setting..
  - A lack of training data that covers the operating envelope
  - Relationships that change depending on operational conditions
  - Sparse sensor networks with reliability problems

✍ E. J. Cross, S. J. Gibson, M. R. Jones, D. J. Pitchforth, S. Zhang, and T. J. Rogers. "Physics-Informed Machine Learning for Structural Health Monitoring." In *Structural Health Monitoring Based on Data Science Techniques*, pp. 347-367. Springer, Cham, 2022.

6 / Dynamics Research Group, University of Sheffield



# A first example – bridge performance monitoring



✎ S. Zhang, T. J. Rogers, and E. J. Cross. Gaussian process based grey-box modelling for SHM of structures under fluctuating environmental conditions. EWSHM 2020.





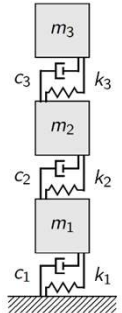
# EPSRC fellowship

## Philosophy

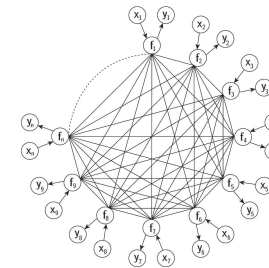
- Consider a grey-box to be a combination of a physics-based model and a data-based model
- Start with the simplest physics that are well understood and can be validated
- Increase flexibility via machine learning/data-driven models

White box / Physics based

- Structured
- Physically insightful



Grey-box



Black box / Machine learning

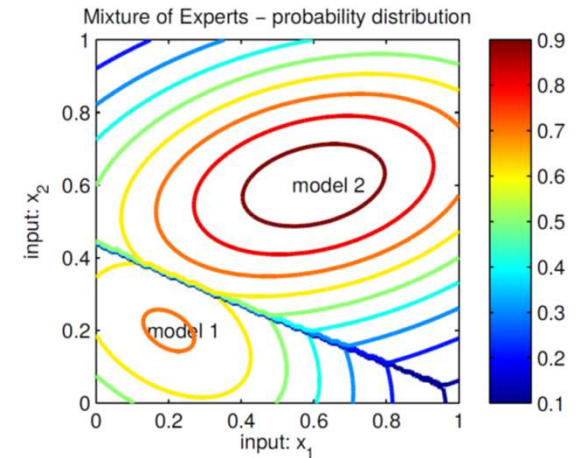
- Flexible
- Data driven





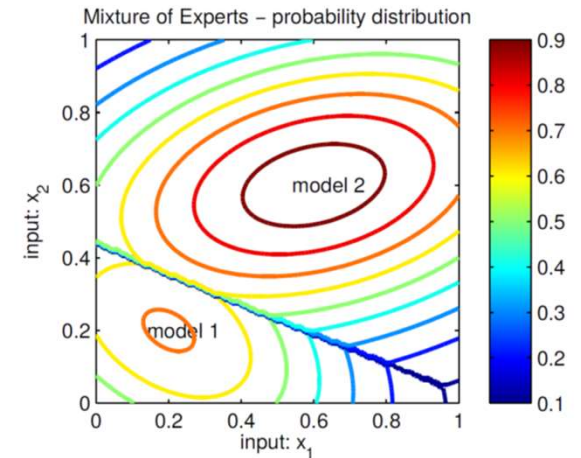
# Fundamental research questions

- Model formulation/architecture
- Optimal balance of explanatory power between the two components
- How to validate these models
- How to incorporate predictive distributions for use in a predictive maintenance strategy



# Fundamental research questions – reordered

- Model formulation/architecture
- How to incorporate predictive distributions for use in a predictive maintenance strategy
- How to validate these models
- Optimal balance of explanatory power between the two components



# Possible physics-informed ML architectures

$$\mathbf{y}_p = \overbrace{f(X)}^{\text{White Box}} + \overbrace{\epsilon(X)}^{\text{Black Box}}$$
$$\mathbf{y}_p = g(X, \underbrace{f(X)}_{\text{White Box}}) \quad \text{Black Box}$$

✍ E. J. Cross, S. J. Gibson, M. R. Jones, D. J. Pitchforth, S. Zhang, and T. J. Rogers. "Physics-Informed Machine Learning for Structural Health Monitoring." In *Structural Health Monitoring Based on Data Science Techniques*, pp. 347-367. Springer, Cham, 2022.

🔗 [https://drg-greybox.github.io/research\\_overview/](https://drg-greybox.github.io/research_overview/)

Analytic models/first principles  
Numerical models FE models/CFD  
Empirical models

Bias correction

Residual Modelling  
Physics-based mean functions

Manipulation of black-box inputs  
Input augmentation

Expressive kernels

State-space formulations 🏗

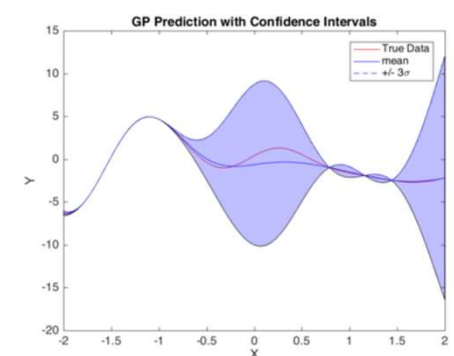
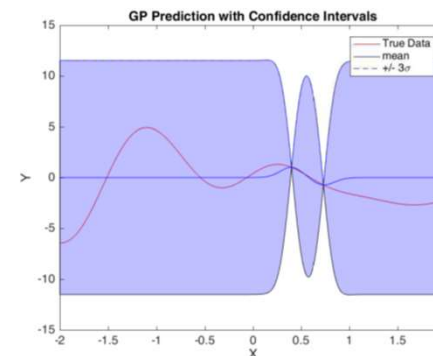
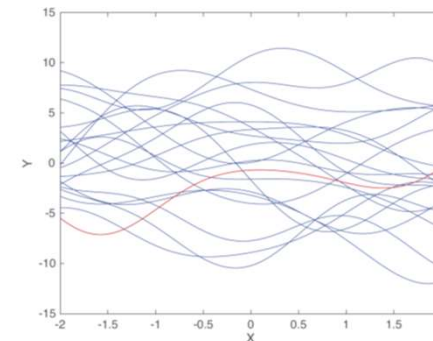
Constrained machine learners

GPs  
Neural networks, etc

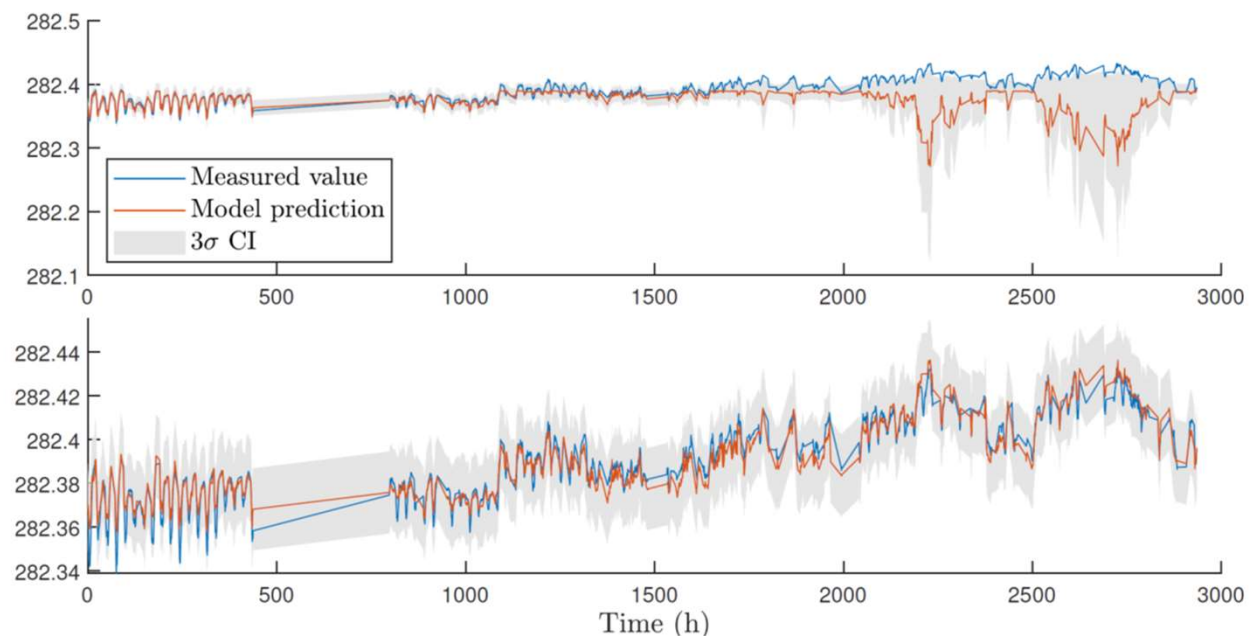


# Gaussian Process Regression

- Gaussian processes (GPs) are employed as a nonparametric Bayesian approach to regression and classification problems.
- GP regression considers a family of functions that fit to a training data set and provides a predictive distribution as opposed to a single crisp prediction for a given input.
- Confidence intervals for each prediction readily available.



# A first example – bridge performance monitoring

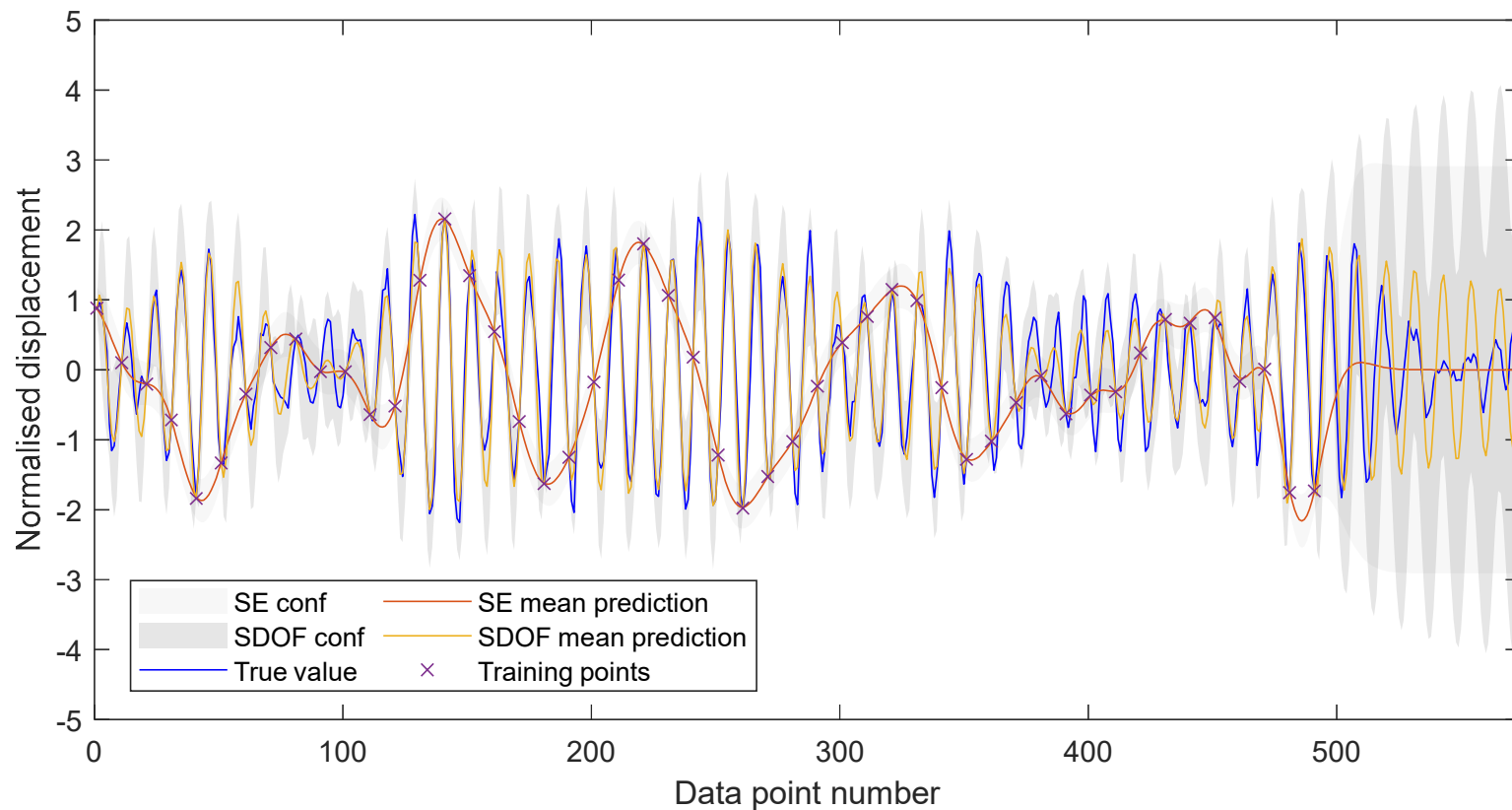


✎ S. Zhang, T. J. Rogers, and E. J. Cross. Gaussian process based grey-box modelling for SHM of structures under fluctuating environmental conditions. EWSHM 2020.



# Physics-derived covariance functions

$$m\ddot{y}(t) + c\dot{y}(t) + ky(t) = F(t) \quad \longrightarrow \quad \phi_Y(\tau) = \frac{\sigma^2}{4m^2\zeta\omega_n^3} e^{-\zeta\omega_n|\tau|} \left( \cos(\omega_d\tau) + \frac{\zeta\omega_n}{\omega_d} \sin(\omega_d|\tau|) \right)$$



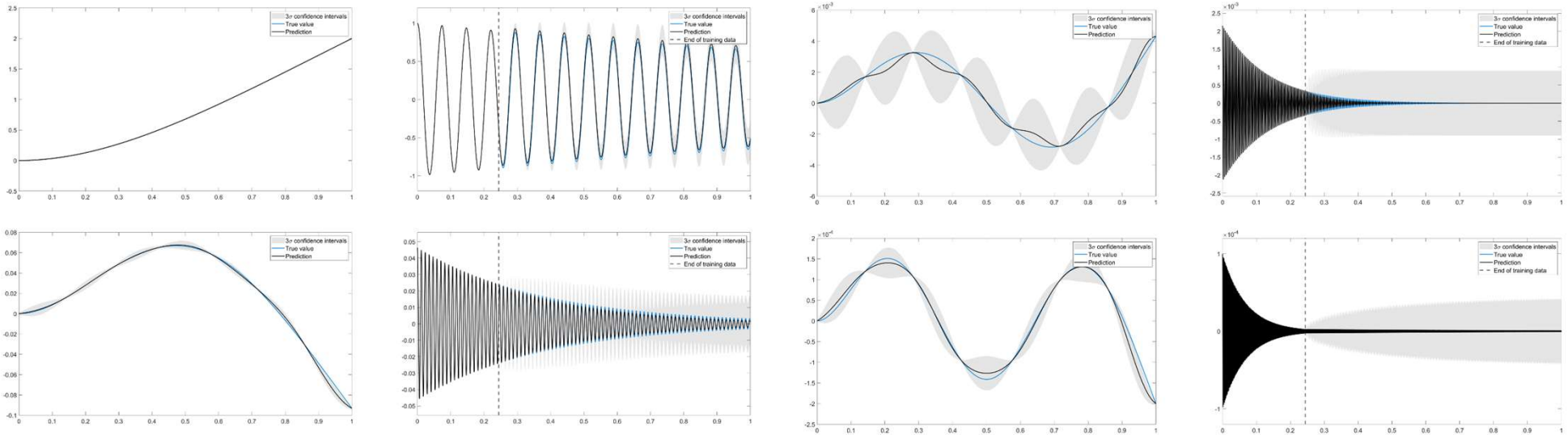
✎ Cross, Elizabeth J., and Timothy J. Rogers. "Physics-derived covariance functions for machine learning in structural dynamics." *IFAC, System Identification: learning models for decision and control - 19th SYSID 2021* 54.7 (2021): 168-173.



# Addressing partial knowledge

- Often we will only have partial knowledge of a system
- In this case the flexibility of the GP formulation allows us to capture that partial knowledge

$$f(\mathbf{x}) \sim \mathcal{GP}(\mu_P, k_P + k_{ML})$$



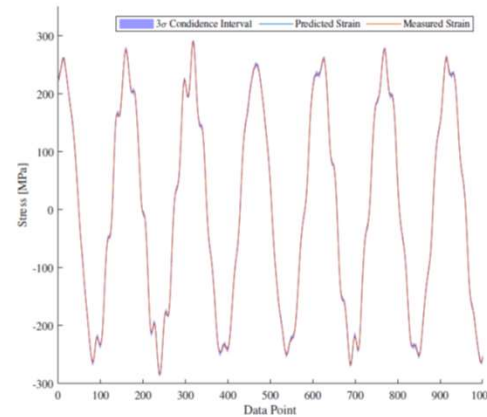
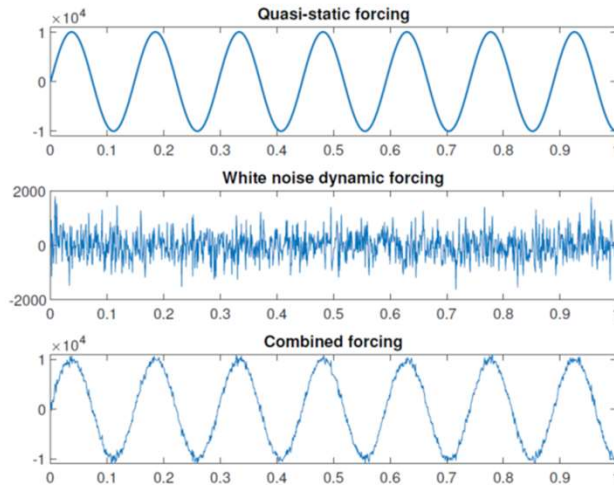
✍ D. J. Pitchforth, T. J. Rogers, U. T. Tygesen, E. J. Cross, "Incorporation of partial physical knowledge within Gaussian processes", In the proceedings of ISMA 2022

✍ D. J. Pitchforth, T. J. Rogers, U. T. Tygesen, E. J. Cross, "Physically informed kernels for wave loading prediction", SHMII11, the 11th International Conference on Structural Health Monitoring of Intelligent Infrastructure

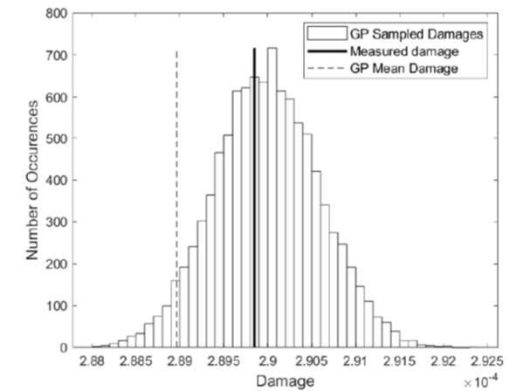




# Addressing partial knowledge & using it in a probabilistic setting



(a) Posterior



(b) Damage distribution



✎ S. Gibson, T. J. Rogers, and E.J. Cross "Data-driven strain prediction models and fatigue damage accumulation," in *Proceedings of ISMA2020, 2020*, pp. 3067–3075

✎ S. Gibson, T. J. Rogers, and E.J. Cross "Integrating physical knowledge into Gaussian process regression models for probabilistic fatigue assessment" *EWSHM 2022 – presenting Wednesday 11.30 Aula 6*

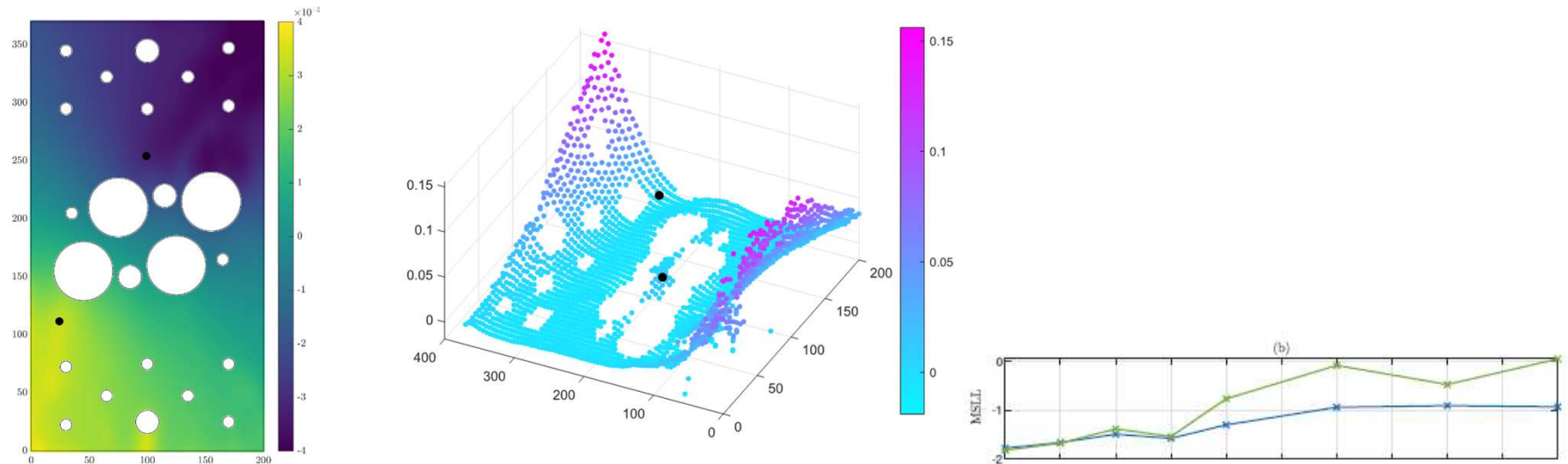
See also

✎ M.R Jones, T. J. Rogers, and E.J. Cross "Physical covariance functions for dynamic systems with time dependent parameters" *EWSHM 2022 – presented yesterday*



# A physics-informed machine learning approach for CM

## Constrained machine learners for damage localisation with acoustic emission



Interpolative model for crack localisation. Constrained learner allows accurate prediction with few training points.

✍ Matthew R Jones, Timothy J Rogers, Keith Worden, and Elizabeth J Cross. A Bayesian methodology for localising acoustic emission sources in complex structures. *Mechanical Systems and Signal Processing*, 163:108143, 2022.

✍ M. R. Jones, T. J. Rogers, and E. J. Cross. Constraining Gaussian processes for physics-informed acoustic emission mapping. *Submitted*, available on arXiv, 2022.

✍ M. R. Jones, T. J. Rogers, I. E. Martinez, and E. J. Cross. Bayesian localisation of acoustic emission sources for wind turbine bearings. In *In Proceedings of SPIE: Health Monitoring of Structural and Biological Systems XV*. 2020



# Ongoing and future challenges



# The path to validation

## Lab for Verification and Validation (LVV)

Our latest facility enables controlled testing of components and full scale structures in a range of environments

- 3 environmental chamber
- Wind and rain simulation
- 3 x 2 m shake table
- 12m wave tank

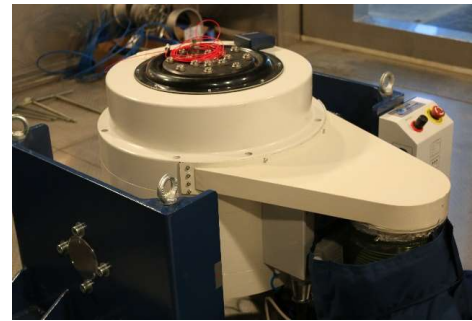


**European Union**  
European Regional  
Development Fund

**EPSRC**  
Engineering and Physical Sciences  
Research Council



# Laboratory for Verification and Validation



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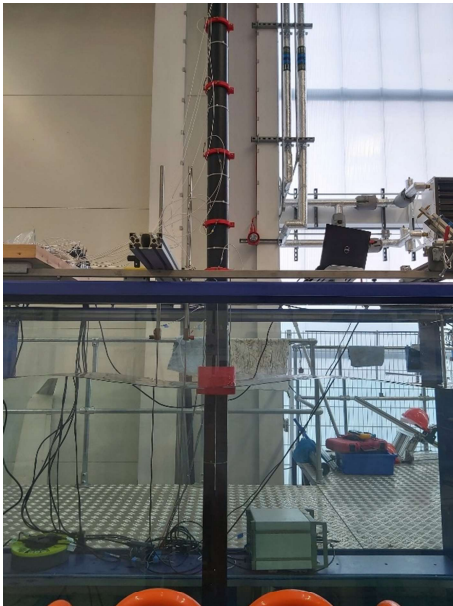
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# Wave loading prediction – extrapolation study



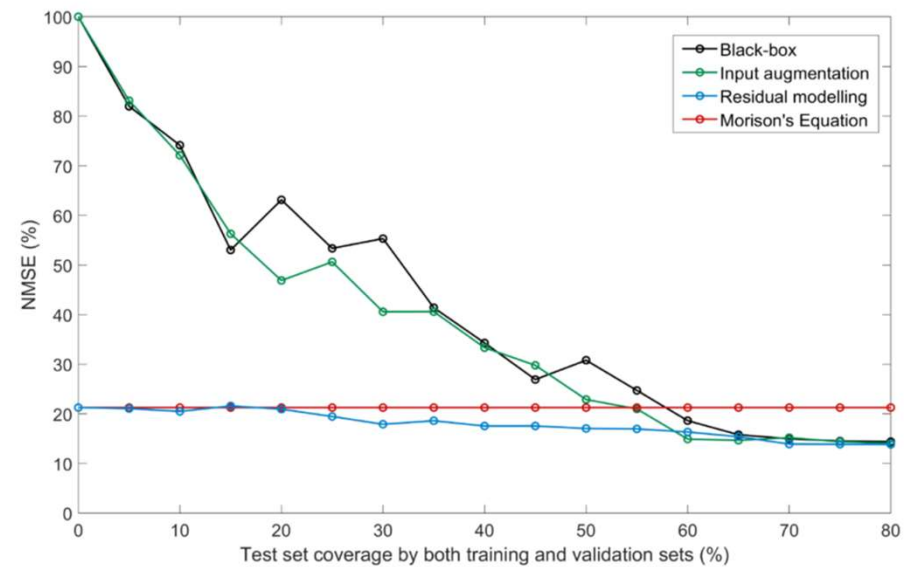
Residual model

$$\mathbf{y}_p = \overbrace{f(X)}^{\text{White Box}} + \overbrace{\epsilon(X)}^{\text{Black Box}}$$

Input augmentation

$$\mathbf{y}_p = g(X, \overbrace{f(X)}^{\text{Black Box}})$$

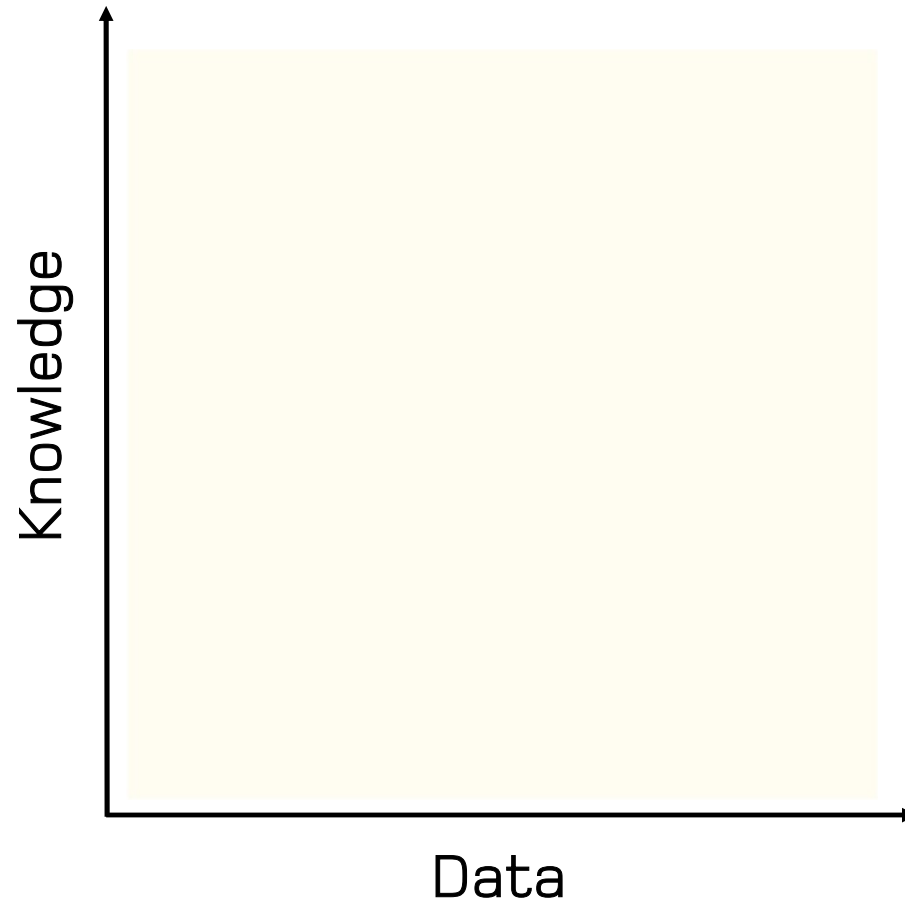
White Box



✎ Pitchforth, Daniel J., Timothy J. Rogers, Ulf T. Tygesen, and Elizabeth J. Cross. "Grey-box models for wave loading prediction." *Mechanical Systems and Signal Processing* 159 (2021): 107741.

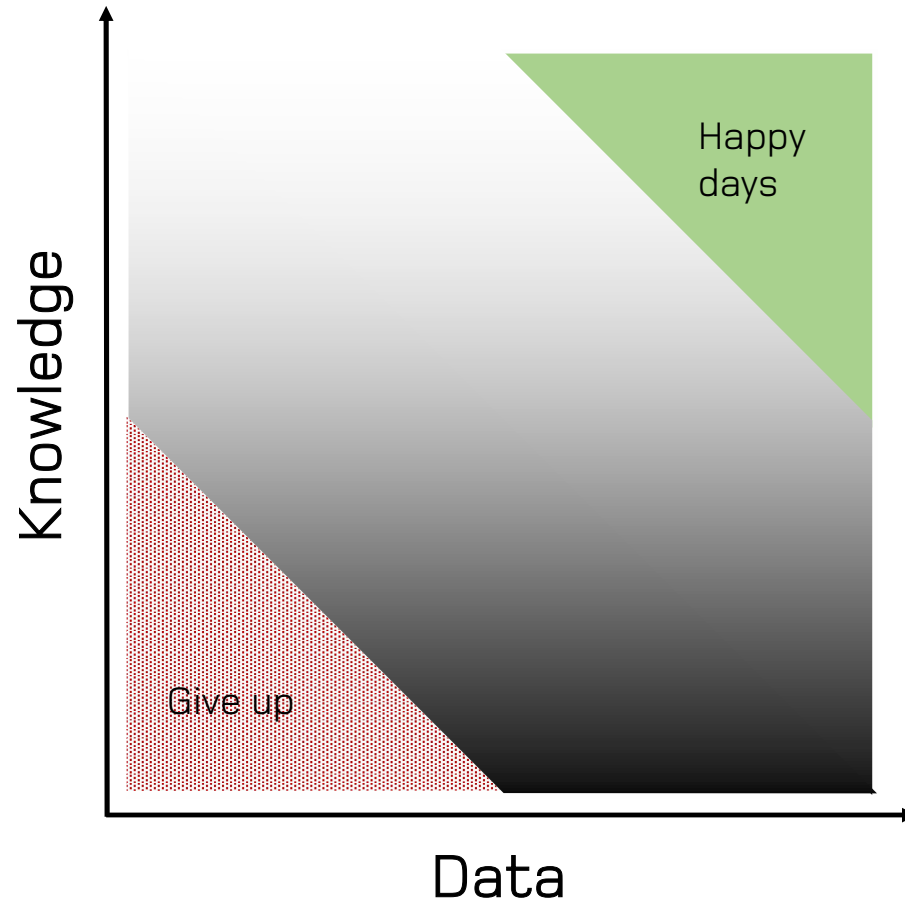


# Optimising explanatory balance between data and physics

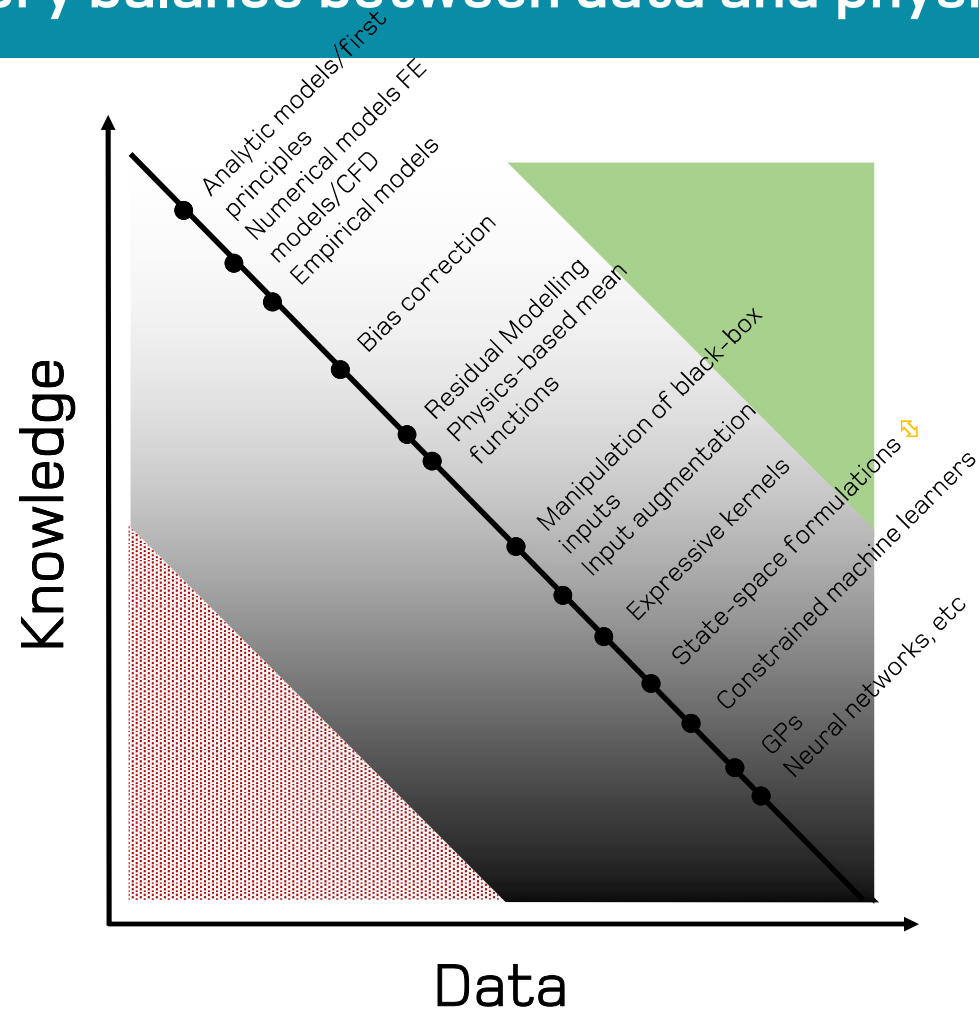




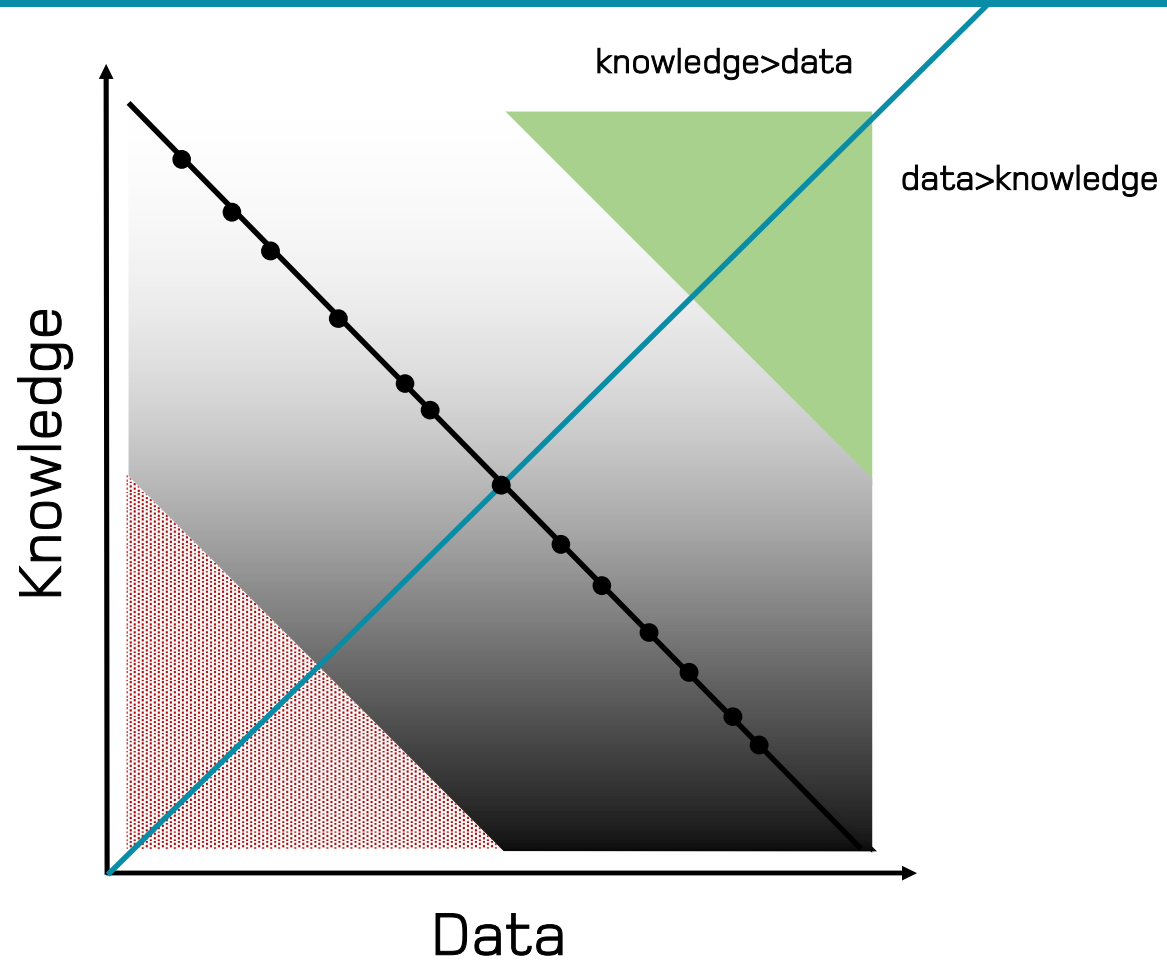
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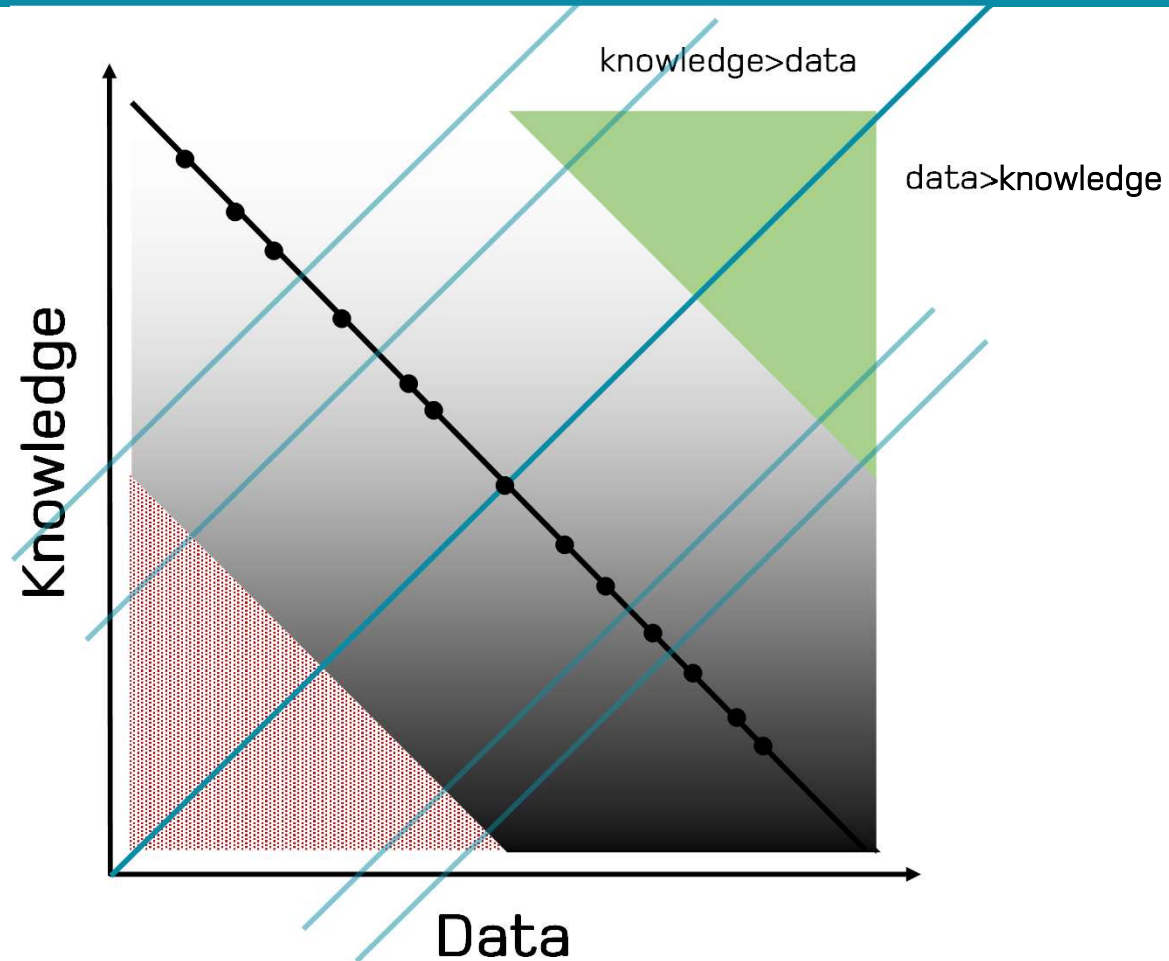


# Optimising explanatory balance between data and physics



# Optimising explanatory balance between data and physics

Partitioning the space



# Conclusions and final discussion

## Summary

- Data-driven/ML methods critical for SHM
- Motivation for a physics-informed approach comes from the reality of how available data are and a wish to incorporate our engineering knowledge
- Bayesian approach provides means of building in physical insight and provides probabilistic assessment that is interpretable
- Ongoing significant challenges, as described, are balancing model components and model validation



# Conclusions and final discussion

## Fair warning

- Beware the hype train, these are not panaceas
- Assessing and expressing your prior knowledge in an appropriate form can be very challenging

## When is it useful?

- When you can prescribe some behaviour not evident from your training data
- When you need a flexible but parsimonious model
- When you want interpretability
- When your knowledge simplifies the learning problem





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Thanks for listening 

## Publications

### Overviews of grey-box models

Physics-informed machine learning for structural health monitoring  
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Book Chapter  
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### Semi-physical modelling



[drg-greybox.github.io/](https://drg-greybox.github.io/)



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